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On the hysteretic to input energy ratio

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ABSTRACT

The hysteretic to input energy ratio E_H/E_I has been studied in a parametric study of SDOF systems. The most important structural parameters were varied and ground motions of very different characteristics were taken into account. When using viscous damping proportional to the instantaneous stiffness, the maximum hysteretic to input energy ratio amounted to 0.9 and 0.8 in the case of 2 and 5 per cent damping, respectively. These values correspond to systems with the natural period shorter than the predominant period of ground motion. They are practically independent of the hysteretic behaviour, the strength (with the exception of very high strength) and the input motion. In the case of a system with the natural period longer than the predominant period of the input motion, E_H/E_I can be approximately determined as a function of the strength of a system and of the characteristics of the input motion using simple formulae proposed in the paper.

INTRODUCTION

The aseismic design philosophy for usual building structures relies strongly on energy dissipation. The energy input to a structure subjected to strong ground motion is dissipated in part by inelastic deformations (hysteretic energy E_H) and, in part, by viscous damping which represents miscellaneous damping effects other than inelastic deformation (damping energy). Only E_H is assumed to contribute to structural damage.

Different investigators have studied input energy (E_I) and proposed procedures or formulae for determining it quantitatively. A very brief review is given in the next chapter. This paper deals predominantly with the ratio of the hysteretic energy to the input energy. This ratio represents a convenient parameter for determination of hysteretic energy, provided that input energy is known.

A few investigators have studied the E_H/E_I ratio recently. Zahrah and Hall (1982) have found out that the proportion of input energy dissipated by yielding increases as viscous damping decreases and as the displacement ductility of a system increases. Similar observations have been made by Akiyama (1985). Based on these observations he proposed a formula for determining E_H/E_I as a function of viscous damping and strength rather than ductility. As a rough approximation, Akiyama proposed a simplified equation, neglecting the influence of strength. Another expression for E_H/E_I as a function of viscous damping and cumulative ductility was proposed by Kuwamura and Galambos (1989). Nariyuki et al (1989) have observed that the relation E_H/E_I versus period is not influenced by the difference in the earthquake ground motion provided that the periods are properly scaled. In this paper results of a parametric study of single-degree-of-freedom (SDOF) systems are presented. Based on these results simple formulae for determining the E_H/E_I ratio are proposed.

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INPUT ENERGY

From research carried out by different investigators (Akiyama 1985, Zahrah and Hall 1982, Fajfar et al 1989) is known that maximum input energy, which is imparted to systems with fundamental periods in the vicinity of the predominant period of the ground motion, is a stable parameter. It is only scarcely dependent on the strength of the structure, on hysteresis and on damping. A shift of the input energy curves to the shorter periods occurs as the strength and/or the "fatness" of the hysteresis loops decrease. The shift can be physically explained by a change in the effective period due to inelastic behaviour. It is larger in the case of strongly non-linear behaviour, typical for a system with lower strength and hysteresis with low "fatness". When a proper scaling of the accelerograms is used, the maximum input energy is not very dependent on ground motion, either (Fajfar et al 1989, 1990).

Formulae for the numerical calculation of input energy as a function of ground motion parameters have been proposed by Kuwamura and Galambos (1989), Fajfar et al (1989), and Uang and Bertero (1990). A preliminary comparative statistical study performed by the authors has indicated that the best prediction of the input energy can be generally obtained by a corrected version of the formula proposed by Kuwamura and Galambos. This formula, however, defines input energy as a function of the integral of the squared ground acceleration which may be difficult to predict in a design procedure. According to Fajfar et al, input energy per unit mass can be estimated as a function of the peak ground velocity v_g and the duration of strong ground motion t_D (defined according to Trifunac and Brady 1975, in seconds)

$$E_I / m = 2.2\, t_D^{0.5}\, v_g^{2} \tag{1}$$

Eq. 1 yields acceptable results for the majority of different types of ground motion. Only two of the basic ground motion parameters which can be routinely predicted in the design procedure are included in the formula. The input energy according to Eq. 1 represents the maximum input energy corresponding to the region in the vicinity of the predominant period of the ground motion. The value of this period can be estimated by a formula proposed by Heidebrecht (1987)

$$T_1 = 4.3\, v_g / a_g \tag{2}$$

where a_g is the peak ground acceleration. Usually it is conservatively assumed (Akiyama 1985, Kuwamura and Galambos 1989) that input energy is constant in the whole medium- and long-period ranges (i.e. for all periods longer than T_1). Uang and Bertero (1990) called attention to the difference between "absolute" and "relative" energy formulations, which is important in the very short and very long period ranges. In this study, the "relative" energy formulation has been used. The relative input energy is defined as the work done by the equivalent force (mass multiplied by ground acceleration) on the equivalent fixed-base system. The effect of the rigid body translation of the structure is neglected.

PARAMETRIC STUDY

To find general purpose expressions for hysteretic to input energy ratio, E_H/E_I, it is necessary to identify the influence of the most important strong motion and structural parameters on the response of a structure. In this study, input ground motion, as well as initial stiffness (period), strength, hysteretic behaviour and damping of SDOF systems were varied. All values of energies were determined at the time equal to the time at the end of the ground motion plus two initial periods of the system. The study represents a continuation of a study on seismic demand performed by the same authors. A detailed description of the parameters can be found in (Fajfar et al 1989). Here, only a very brief overview will be given.

The influence of input motion has been studied using five different groups of records in order to take into account ground motions of basically different types. Standard records from California and records from Montenegro, Yugoslavia, 1979, are representative for "standard" ground motion. The main characteristics of the Friuli, Italy, 1976, and Banja Luka, Yugoslavia, 1981, records is the short duration of the strong ground motion.

The predominant periods of these records are short and fairly narrow-banded, and the peak ground velocity to peak ground acceleration ratios are small. The 1985 Mexico City records represent ground motions of very long duration, with long predominant periods and high peak ground velocity to peak ground acceleration ratios. Totally, 40 horizontal components of records obtained at 20 different stations have been used.

The period range from 0.1 to 2.5 s was considered. The value of the strength parameter η, which is defined as the yield resistance F_y divided by the mass of the system and by the peak ground acceleration a_g

$$\eta = F_y / (m\, a_g) \tag{3}$$

was varied from 1.5 to 0.2. Both mass-proportional viscous damping and viscous damping proportional to instantaneous stiffness (2 and 5 per cent) were used. Altogether, eight different hysteresis models were investigated. Six of them simulate predominantly flexural behaviour: elastic-plastic, bilinear, the Q-model and three variants of Takeda's model with trilinear envelope (they differ according to unloading stiffness and envelope shape). The shear behaviour is simulated by two variants of the shear-slip model.

The main results of the study are presented in Figs. 1 to 4. The hysteretic to input energy ratio is plotted as a function of the initial (elastic) period of the system. In different figures the influence of damping, strength, hysteresis model and input motion are shown. The results of the parametric study of different structural parameters are presented as mean values from 20 "standard" records (the U.S.A. and Montenegro groups). The medium-period range for these records is roughly between periods 0.5 s and 2.0 s. The inelastic system with $\eta = 0.6$, Q-hysteresis and 5 per cent of damping proportional to instantaneous stiffness was chosen as the basic "average inelastic system" which is supposed to represent an average reinforced concrete structure designed according to the codes. In this system, strength, hysteresis and damping were varied one by one.

As shown in the figures, the E_H/E_I ratio has generally its peak values in the short-period range, where the periods are shorter than the predominant period of ground motion T_1. In the medium- and long-period range, where the periods are longer than T_1, the E_H/E_I ratio decreases as the period increases. The zero value of E_H/E_I, indicating elastic behaviour, is reached at the period which will be denoted by T_o.

One of the most important parameters influencing the E_H/E_I ratio is damping. The result of a decrease in viscous damping is an increase of the E_H/E_I ratio in the whole period range (Fig. 1). It is important to realize that the E_H/E_I ratio is strongly influenced by the mathematical modeling of viscous damping. Damping may be assumed to be proportional either to mass or to stiffness. In linear analysis both approaches yield the same results. In nonlinear analysis, however, the stiffness degrades with damage. Consequently, damping related to the instantaneous stiffness tends to decrease, and damping related to the mass tends to increase with degrading of stiffness. In the case of inelastic analysis mass-proportional damping dissipates more energy and is thus more effective in reducing hysteretic energy. This tendency is larger for a system in the short-period range. Systems with longer period experience less stiffness degradation and the influence of the damping model is less important. Similar conclusions have been obtained by Otani (1981). He stated that it was not probable to expect mass-proportional damping in a real reinforced

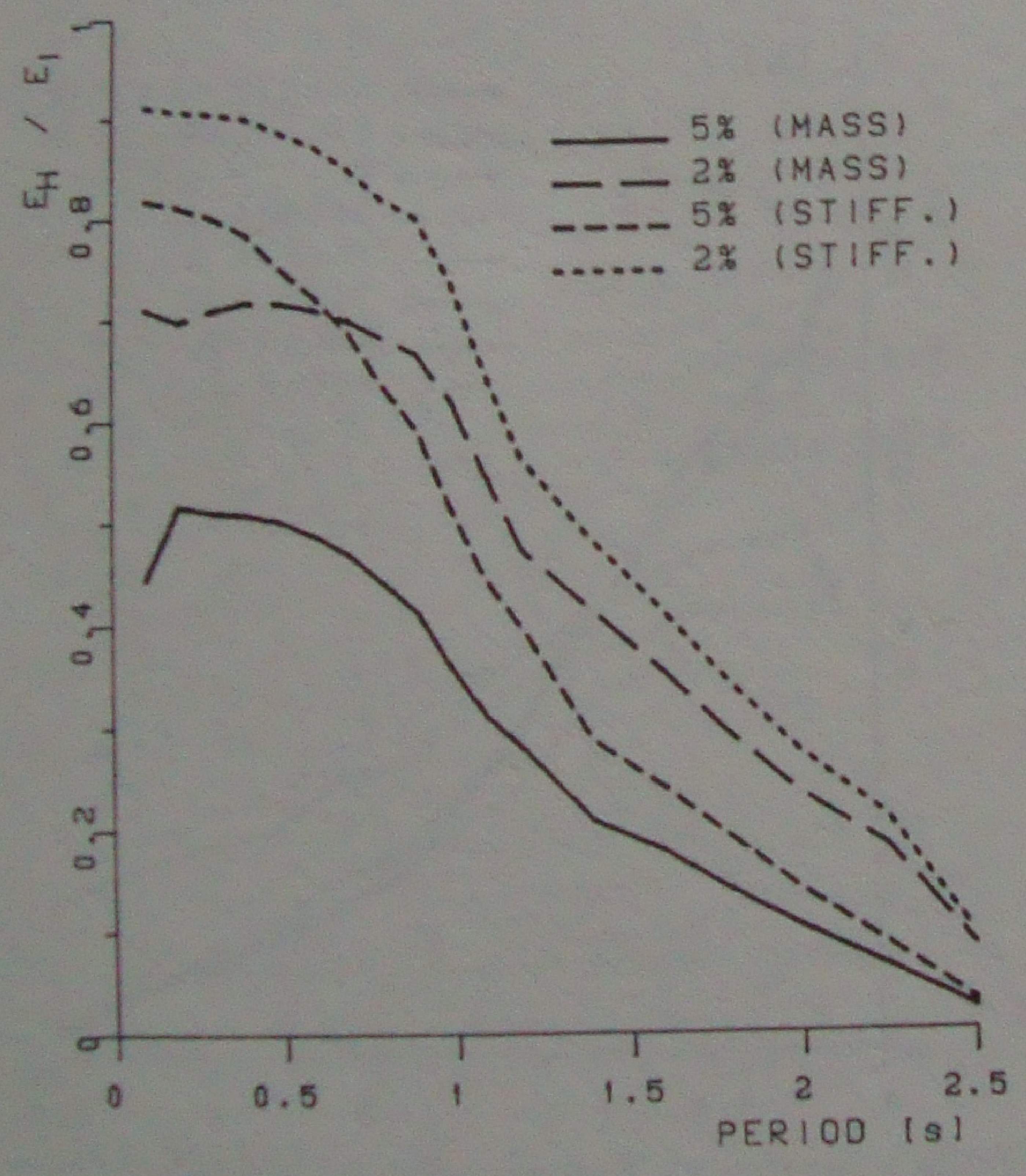

Figure 1. E_H/E_I versus T. Influence of damping

concrete structure. Consequently, in our study mainly the stiffness-proportional damping was used. This decision was supported by the observation that the stiffness-proportional damping eliminated the influence of the hysteretic behaviour to a great extent (Fig. 2).

The authors believe that the concept of viscous damping in nonlinear analysis is questionable. In the nonlinear range, it seems to be reasonable to define the damping, which includes the effect of energy dissipation other than hysteretic energy, as a portion of the input energy. More experimental and analytical work on this subject is needed.

The influence of different hysteretic behaviour can be observed in Fig. 2. It can be seen that the type of the hysteretic behaviour is very important in the case of the mass-proportional damping (Fig. 2b) and has a surprisingly small influence in the case of the instantaneous-stiffness-proportional damping (Fig. 2a). An important difference can be observed only in the case of Takeda's hysteretic rules where trilinear envelopes were used. The period T in the figures corresponds to the initial stiffness, i.e. the stiffness before cracking in the case of a trilinear envelope. If an equivalent stiffness or the yield stiffness was considered at the horizontal axis, the difference between the two sets of curves, corresponding to bilinear and trilinear envelopes, would disappear.

The influence of the strength of a system can be seen in Fig. 3. Usually it has been believed that E_H/E_I decreases with an increase in strength (or with a decrease in ductility). It was surprising to find out that this is not always the case. The maximum E_H/E_I ratio is, with exception of a system with very high strength ($\eta > 1$), a reasonably stable quantity practically independent of the strength of a system. The decrease of the E_H/E_I ratio in the medium- and long-period range, however, strongly depends on the strength. A system with low strength experiences large inelastic excursions and dissipates more hysteretic energy than a system with high strength and the same stiffness.

Mean values of E_H/E_I for different groups of records are shown in Fig. 4. It can be seen that the maximum E_H/E_I values are more or less the same for all groups of records. The values of the predominant period T_1 and of the period T_0, however, strongly depend on the type of ground motion.

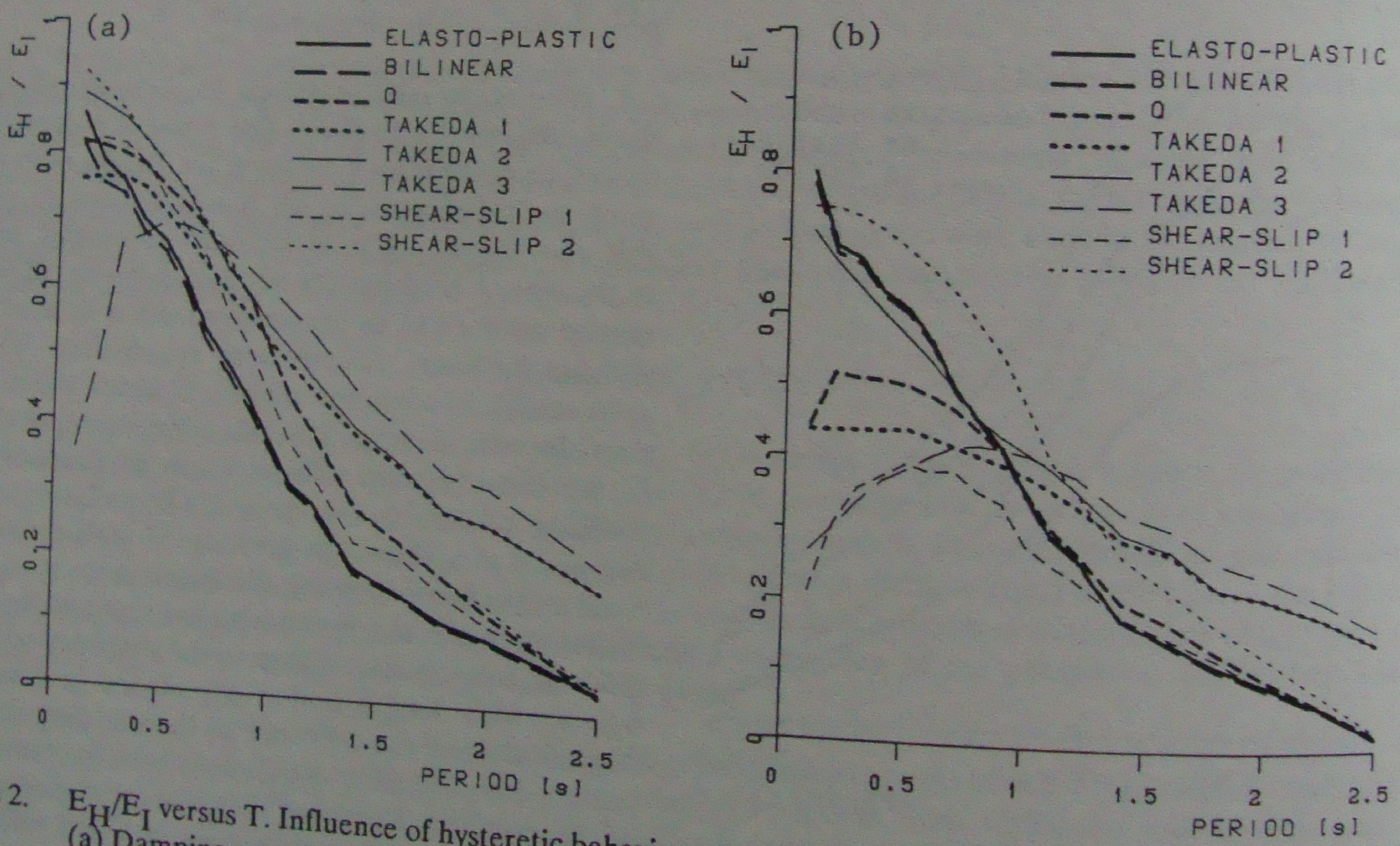

Figure 2. E_H/E_I versus T. Influence of hysteretic behaviour.
(a) Damping proportional to instantaneous stiffness. (b) Damping proportional to mass

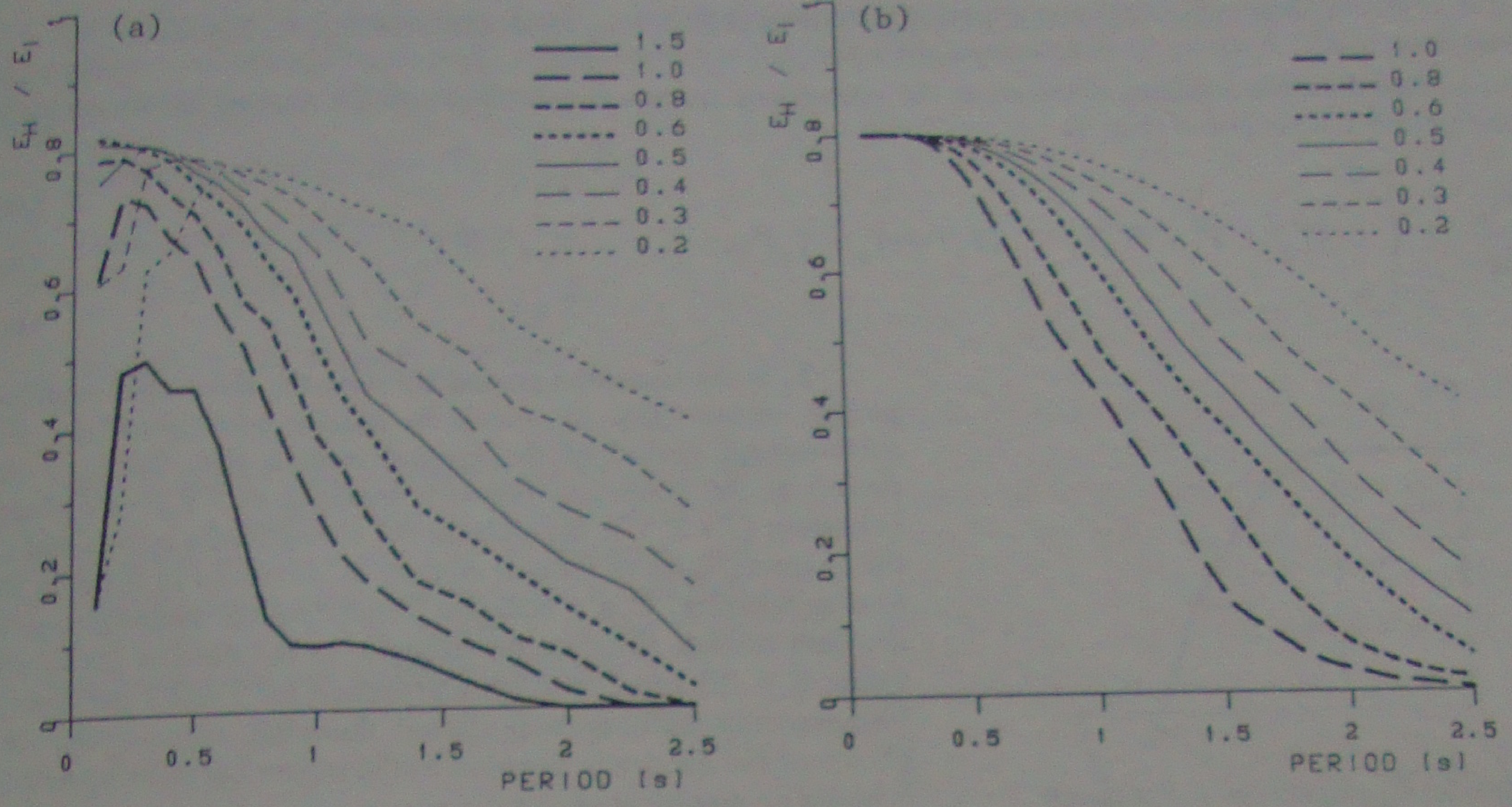

Figure 3. E_H/E_I versus T. Influence of the strength of a system.
(a) Mean curves for 20 "standard" records
(b) Mean of curves determined according to Eq. 4 for "standard" records

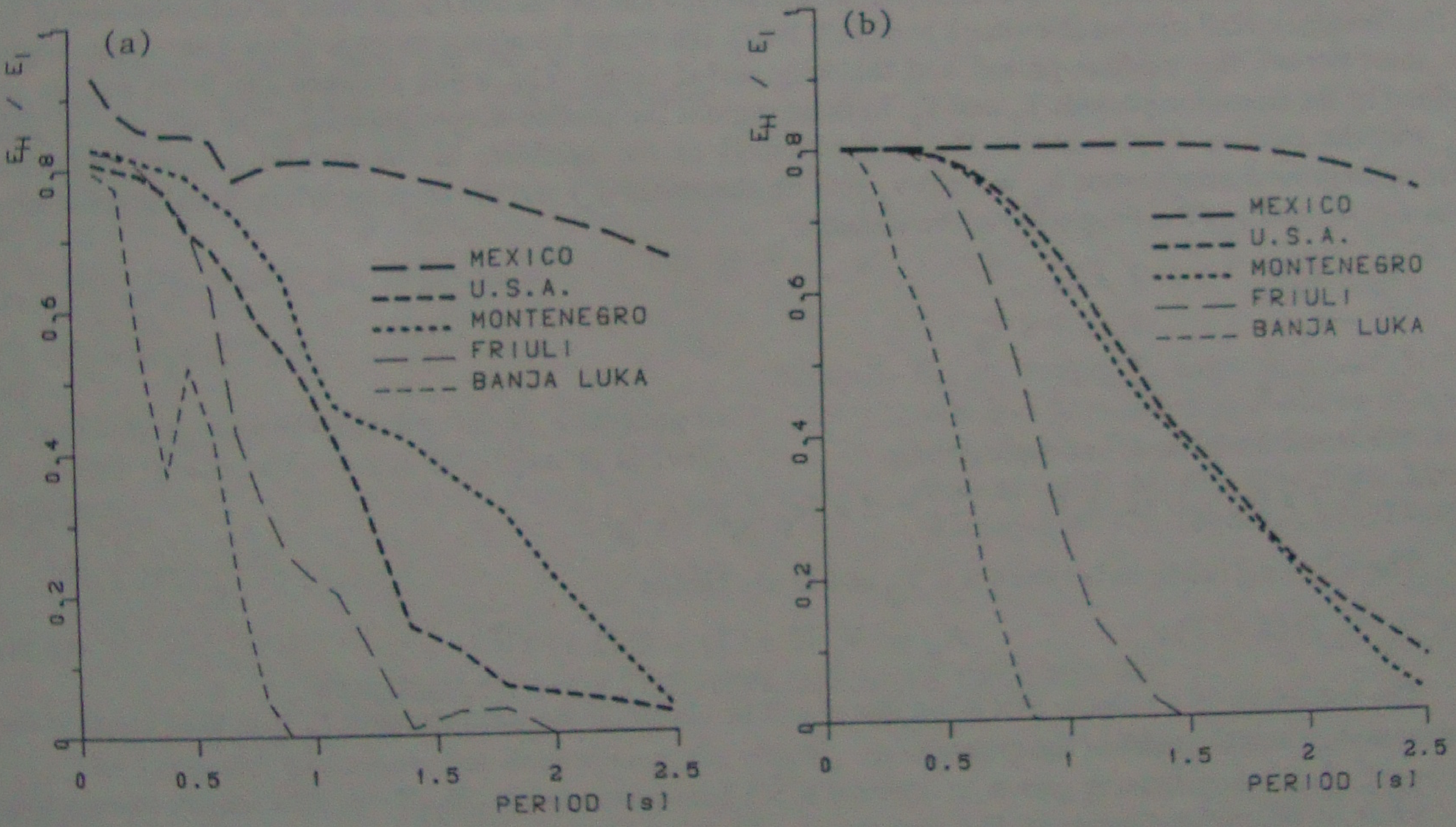

Figure 4. E_H/E_I versus T. Influence of input motion.
(a) Mean curves for different group of records
(b) Mean curves determined according to Eq. 4 for different groups of records

APPROXIMATE PROCEDURE FOR DETERMINATION OF THE E_H/E_I RATIO

Based on observations obtained in the parametric study a simple approximate relation between E_H/E_I and period T is proposed (Fig. 5)

$$\frac{E_H}{E_I} = \begin{cases} C & T \le T_1 \quad (4a) \\ C\,(T_o^2 - T^2 - 2T_oT_1 + 2T_1T)/(T_o - T_1)^2 & T_1 \le T \le T_o \quad (4b) \\ 0 & T \ge T_o \quad (4c) \end{cases}$$

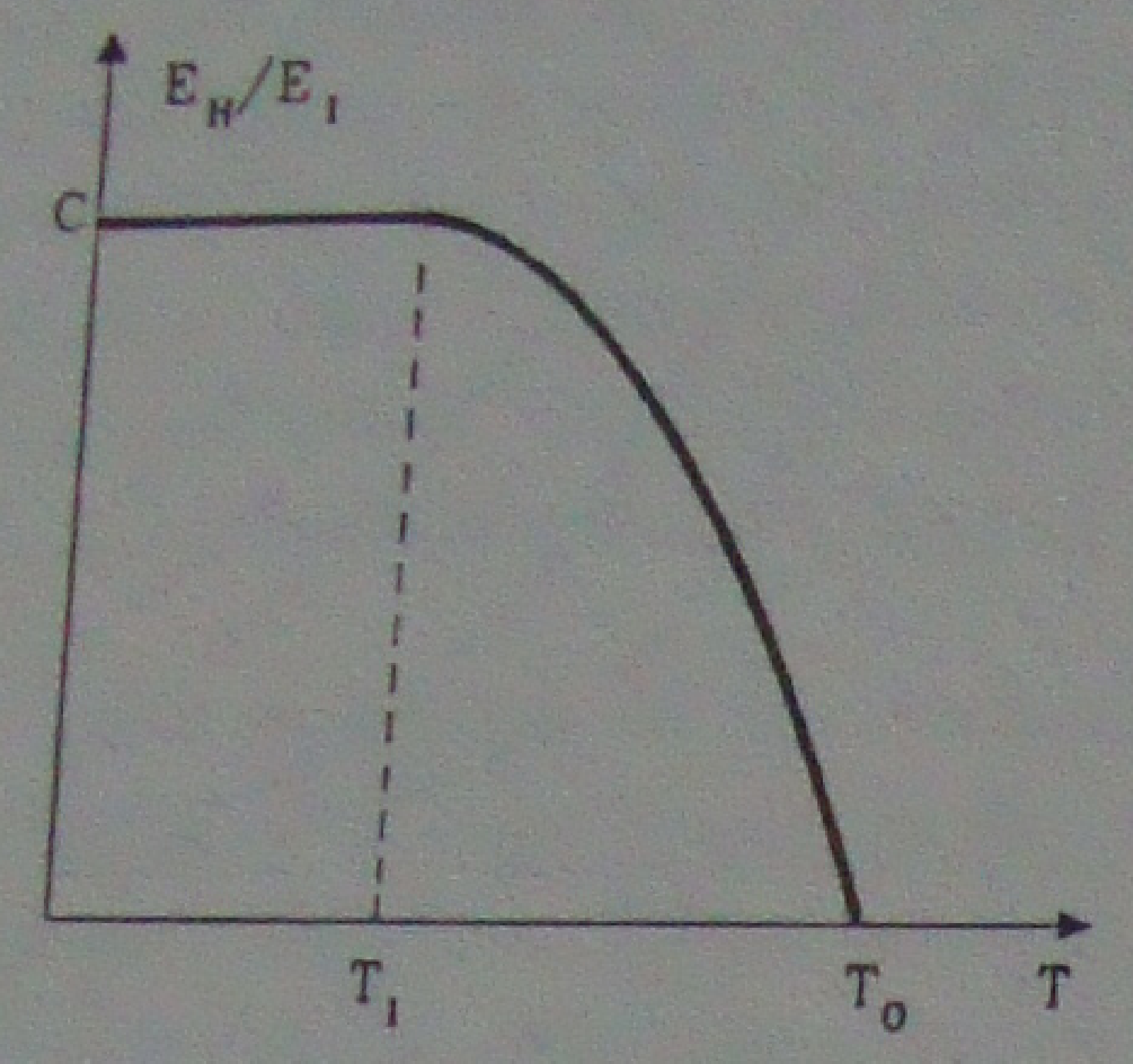

Figure 5. E_H/E_I versus T. Proposed relation

T_1 is the transition period between the short- and medium-period region (Eq. 8) which is usually considered as the predominant period of the ground motion (Eq. 2). T_o is the period at the transition between inelastic ($T < T_o$) and elastic behaviour ($T > T_o$). It depends on the strength of a system and on the characteristics of ground motion, as discussed in detail in the next subchapter. C is a constant depending on damping and, in the case of high strength, on strength. In the case of $\eta \le 1.0$ the values C = 0.8 and C = 0.9 are proposed for 5 and 2 per cent of instantaneous-stiffness-proportional damping, respectively. The choice of the quadratic function in the region between T_1 and T_o is based on empirical observations.

Determination of To

The zero value of the E_H/E_I ratio, indicating elastic behaviour, is reached at the period which was denoted by T_o. The elastic spectra of the Newmark-Hall type can be used in the process of determination of T_o. In the Newmark-Hall spectra (Newmark and Hall, 1982), the whole frequency range is divided into three ranges: the short-period, the medium-period and the long-period range. The limits between the three regions are defined by the transition periods T_1 and T_2. In these regions the pseudo-accelerations A_p, the pseudo-velocities V_p and the relative displacements D are determined as the products of the ground motion parameters (maximum ground acceleration a_g, velocity v_g, and displacement d_g) and the corresponding elastic amplification factors c_a, c_v and c_d (T is the period of the system):

$$A_p = c_a a_g \qquad T \le T_1 \qquad (5)$$

$$V_p = c_v v_g \qquad T_1 \le T \le T_2 \qquad (6)$$

$$D = c_d d_g \qquad T \ge T_2 \qquad (7)$$

The transition periods are expressed by

$$T_1 = 2\pi\,(c_v v_g)/(c_a a_g), \qquad T_2 = 2\pi\,(c_d d_g)/(c_v v_g) \qquad (8,9)$$

The following relations between A_p, V_p and D are known

$$V_p = D\,(2\pi/T), \qquad A_p = V_p\,(2\pi/T) = D\,(2\pi/T)^2 \qquad (10,11)$$

The behaviour of a bilinear SDOF oscillator will be elastic, if the yield strength F_y is at least equal to the product m A_p (m is the mass of the system)

$$F_y = m\,A_p \qquad (12)$$

or larger. Using Eq. 3, Eq. 12 may be written as

$$\eta\, a_g = A_p \qquad (13)$$

In the derivation of the formula for T_o two possibilities will be considered. It will be assumed that T_o is in the medium- and long-period range, respectively. First, general expressions will be derived. Then, constants proposed by the authors will be employed.

a) Medium-period region ($T_1 \le T_o \le T_2$)

From Eqs. 6, 11 and 13 it follows

$$T_o = (2\pi c_v/\eta)(v_g/a_g) \tag{14}$$

Using Eq. 8, Eq. 14 can be rewritten

$$T_o = (c_a/\eta)\, T_1 \tag{15}$$

Eqs. 14 and 15 represent two forms of a general formula for T_o, provided that T_o is located in the medium-period region. According to Newmark and Hall (1982) the mean amplification factor c_a amounts to 2.12 in the case of 5 % damping. In the study reported in this paper, the amplification factor

$$c_a = 1.56\, t_D^{0.25} \tag{16}$$

where t_D is the duration of strong ground motion in seconds, was used. Eq. 16 follows indirectly from the results of the parametric study performed by the authors (Fajfar et al 1989) and from Eq. 2.

b) Long-period region ($T_o \ge T_2$)

Formulae derived here will be used, if T_o determined according to Eqs. 14 or 15 is greater than T_2. From Eqs. 7, 11 and 13 it follows

$$T_o^2 = (4\pi^2 c_d/\eta)(d_g/a_g) \tag{17}$$

Using Eqs. 8 and 9, Eq. 17 becomes

$$T_o^2 = (c_a/\eta)\, T_1\, T_2 \tag{18}$$

In our study, c_a according to Eq. 16, T_1 according to Eq. 2, and T_2 according to Eq. 19 (Fajfar et al 1989) were used.

$$T_2 = (13/t_D^{0.25})(d_g/v_g) \tag{19}$$

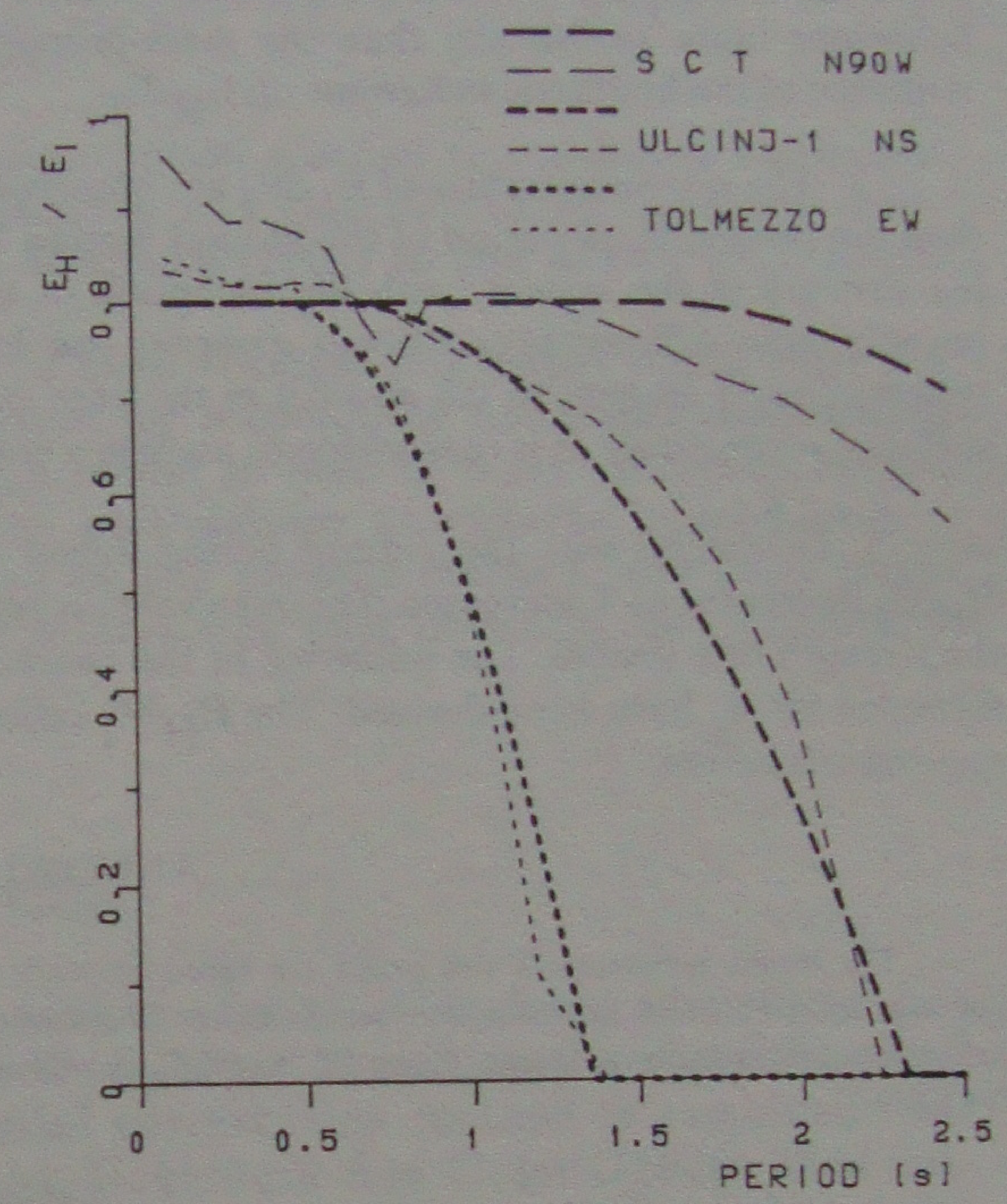

Figure 6. E_H/E_I versus T. Comparison of "exact" relations (thin lines) and curves obtained according to Eq. 4 (thick lines) for three records (SCT, Mexico City 1985; Ulcinj, Montenegro 1979; Tolmezzo, Friuli 1976)

VERIFICATION OF THE APPROXIMATE PROCEDURE

The proposed procedure for determining the E_H/E_I ratio has been verified by a comparison of approximate results with the E_H/E_I values computed for a number of strong ground motions. In Fig. 6 the results for the "average inelastic system" (η = 0.6, Q - hysteresis, 5 % damping) subjected to three very different ground motions are shown. A comparison of mean curves for systems with different strength subjected to 20 "standard" records is shown in Fig. 3. A similar comparison is shown in Fig. 4, where the influence of different groups of records can be examined. A good correlation can be observed in all figures. An exception represents the group of Californian records. The less favourable correlation in this case might be attributed to the choice of the high-pass filter in the correction procedure for accelerograms, which has an important influence on the maximum ground displacement d_g (Fajfar et al 1989) and therefore also on T_o, if located in the long-period region (Eq. 17).

CONCLUSIONS

Based on results of a parametric study of inelastic SDOF systems, the following main conclusions and results have been obtained.

1. The hysteretic to input energy ratio E_H/E_I depends on the amount of viscous damping and also on the modeling of damping. The damping proportional to instantaneous stiffness seems to simulate the real structural behaviour more realistically than the mass-proportional damping. In addition, it practically eliminates the influence of the hysteretic behaviour on E_H/E_I.

2. The maximum values of E_H/E_I are observed in the short-period systems with the natural period shorter than the predominant period of the ground motion T_1. They practically do not depend on the input motion, on the strength of the system (with the exception of a system with high strength $\eta > 1$), and in the case of the instantaneous-stiffness-proportional damping, on hysteretic behaviour. Values of E_H/E_I for usual inelastic systems ($\eta \leq 1$) amount to 0.8 and 0.9 in the case of 5 and 2 per cent damping proportional to instantaneous stiffness, respectively. It is reasonable to assume a constant value of E_H/E_I in the short-period region.

3. In systems with the natural period longer than T_1 (medium- and long-period systems) the value of E_H/E_I decreases as T increases. The zero value is reached at the period T_o. The ratio T_o/T_1 depends mainly on the strength of a system. The influence of the characteristics of ground motion has been also observed. Simple formulae for T_o have been derived. The E_H/E_I ratio between the periods T_1 and T_o can be approximated by a quadratic function.

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